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PULSE TUBE COOLERS WITH AN INERTANCE TUBE: THEORY, MODELING AND PRACTICE

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ABSTRACT

We have studied the advantages to be gained by replacing the orifice of a pulse tube cooler with an inertance tube—a long thin tube that introduces the possibility for additional phase shift between pressure and mass flow in the pulse tube section. A case for using an inertance tube is made by employing an electrical analogy, where the 'inductance' added by the inertance tube allows for an improved power transfer efficiency at the cold end of the pulse tube. Detailed computer modeling of pulse tube systems with inertance tubes confirms these advantages. Comparison between a laboratory cooler with an orifice and with two inertance tubes is presented; the inertance tubes yield dramatic improvements over the use of the orifice.

INTRODUCTION

Pulse tube coolers have a considerable advantage over Stirling coolers because they have no moving parts in the low-temperature region. This leads to much more reliable operation and longer life times for the low-temperature components and also much lower vibration in the cold region. However, Stirling coolers have demonstrated better thermal efficiency than pulse tube coolers because the low-temperature displacer of the Stirling cooler can be driven at a phase that is adjusted to give the best performance. Pulse tube coolers have used either an orifice to create the phase shift between pressure and mass flow in the pulse tube or both an orifice and a double-inlet connection that allows some of the flow to bypass the regenerator and pulse tube. In neither case does the efficiency quite reach that of Stirling coolers.

Recent studies[1,2] have suggested that there is a simple way to generate the phase shift needed to make pulse tube coolers operate as efficiently as Stirling coolers. This is to replace the orifice in the system with an 'inertance' tube. The inertance tube is a long, thin tube that provides a complex impedance at the warm end of the pulse tube rather than a simple resistive impedance that the orifice provides. The inertance tube adds a reactive impedance, analogous to inductance in electrical circuits, that allows the phase between the pressure and mass flow in the pulse tube to be adjusted to an extent that was not previously possible. In principle, it might now be possible to adjust the complex impedance of the inertance tube to achieve the maximum cooling efficiency that the system is capable of.

ELECTRICAL ANALOGY

Considerable insight to the inertance tube can be gained from an analogy to electrical circuits. Consider the relations between electrical current, I, and voltage, V, for the cases of a resistor of resistance, R, an inductor of inductance, L, and a capacitor of capacitance, C:

$$\text{Resistor:}\quad V = I\,R \tag{1}$$

$$\text{Inductor:}\quad V = L\,\frac{d\,I}{d\,t} \tag{2}$$

$$\text{Capacitor:}\quad \frac{d\,V}{d\,t} = \frac{I}{C} \tag{3}$$

Fluid flow

Similar relationships can be found for gas flow and pressure in the elements of a pulse tube cooler. Consider the 1-dimensional momentum conservation equation[3] for the flow in a tube of radius, r:

$$\rho\,\frac{\partial q}{\partial t} = -\,\frac{\partial P}{\partial z} - \frac{\mu\,q}{K_p}\,, \tag{4}$$

where P is the pressure, ρ is the gas density, q is the average velocity in the tube, μ is the viscosity and K_p is the Darcy permeability.

Substituting $q = U/\pi\,r^2$ and rearranging the terms yields:

$$-\,\frac{\partial P}{\partial z} = \frac{\rho}{\pi\,r^2}\frac{\partial U}{\partial t} + \frac{\mu\,U}{\pi\,r^2\,K_p}\,, \tag{5}$$

where U is the volume flow rate. If these parameters are independent of the distance, z, along the tube, then the equation can be integrated along the tube for its entire length, λ, and one gets:

$$-\,\Delta P = \frac{\rho\,\lambda}{\pi\,r^2}\frac{\partial U}{\partial t} + \frac{\mu\,\lambda\,U}{\pi\,r^2\,K_p}\,. \tag{6}$$

If the pressure, P, is taken to be the analog of voltage, V, and the volume flow, U, is taken to be the analog of electrical current, I, then comparison with eqs. (1) and (2) shows that eq. (6) is analogous to a resistance of value

$$R = \mu\lambda/(\pi\,r^2\,K_p\,),\quad (K_p = r^2/8 \text{ for laminar flow}) \tag{7}$$

in series with an inductance of value

$$L = \rho\lambda/\pi\,r^2\,. \tag{8}$$

This 'inductance' analog gets the name of inertance for the gas flow case.

For an isothermal volume, V_t, with volume flow, U, into it, the pressure rise will be:

$$\frac{\partial P}{\partial t} = \frac{P_{av}\, U}{V_t} \; . \tag{9}$$

This is exactly analogous to eq. (3) with a capacitance of value

$$C = V_t \,/ P_{av}. \tag{10}$$

Equations (6) and (9) show that gas flow in tubes and volumes is analogous to electrical current flow in resistors, inductors and capacitances.

Electrical analogy for a pulse tube cooler

In a pulse tube cooler it is important to maximize the cooling efficiency: the ratio of the cooling at the cold end of the pulse tube to the work of the compressor. Figure 1 shows a simple pulse tube cooler with an inertance tube along with an equivalent circuit that approximates the main features of the cooler. In this circuit, all the power dissipation in the system takes place in the resistive elements R_{regen} and R_{inert}. The electrical power that flows through R_{inert} is the analog of the cooling power that flows through the pulse tube and into the inertance tube. The ratio of this power to the sum of this power plus the power dissipated in R_{regen} is the efficiency. If one assumes that all the other parameters are fixed, then only R_{inert} and L_{inert} are to be adjusted. Equations (7) and (8) were used to express R_{inert} and L_{inert} in terms of the length, λ, and the radius, r, of the inertance tube. Figure 2 shows analytical results, using values that are typical for a pulse tube cooler. The contours are constant values

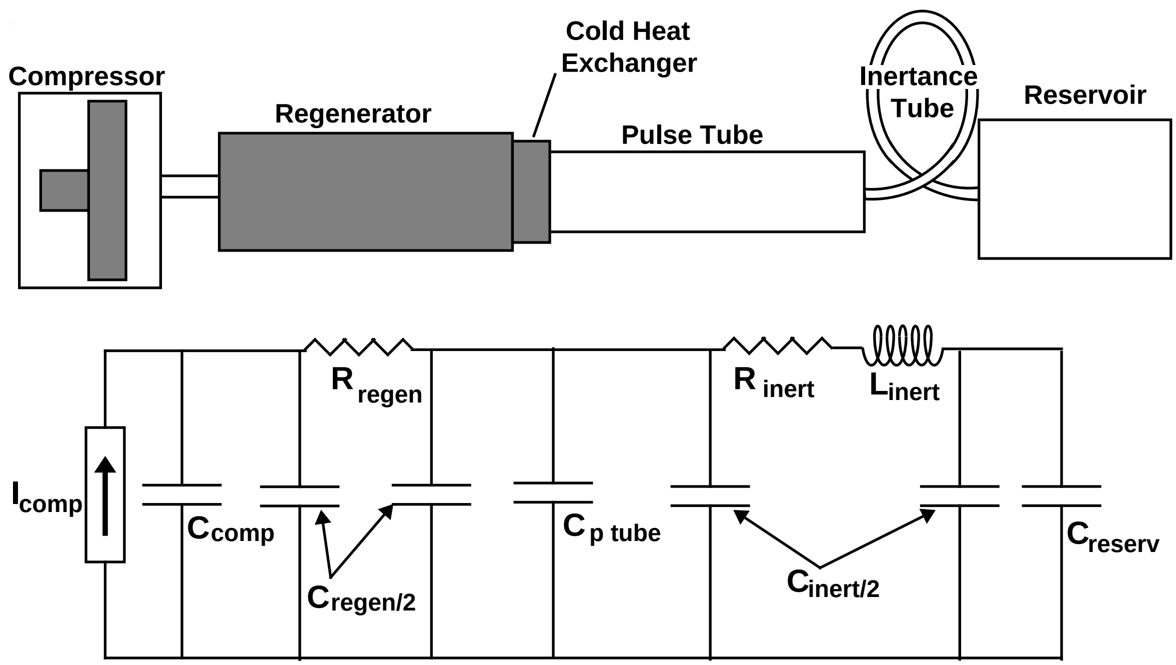

Fig. 1. A simple pulse tube cooler with an inertance tube, and the analogous electrical circuit, using lumped circuit elements. I_{comp} is a current source representing the piston stroke; C_{comp} represents the compressor volume; R_{regen} represents the flow impedance of the regenerator and the two $C_{regen/2}$ are approximations to the distributed volume of the regenerator; $C_{p\ tube}$ represents the volume of the pulse tube; R_{inert} represents the dissipative resistance of the inertance tube; L_{inert} represents the inductive 'inertance' of the inertance tube; $C_{inert/2}$ are approximations to the distributed volume of the inertance tube; and C_{reserv} represents the volume of the reservoir (assumed to be large so that C_{reserv} and the capacitance in parallel with it can be ignored).

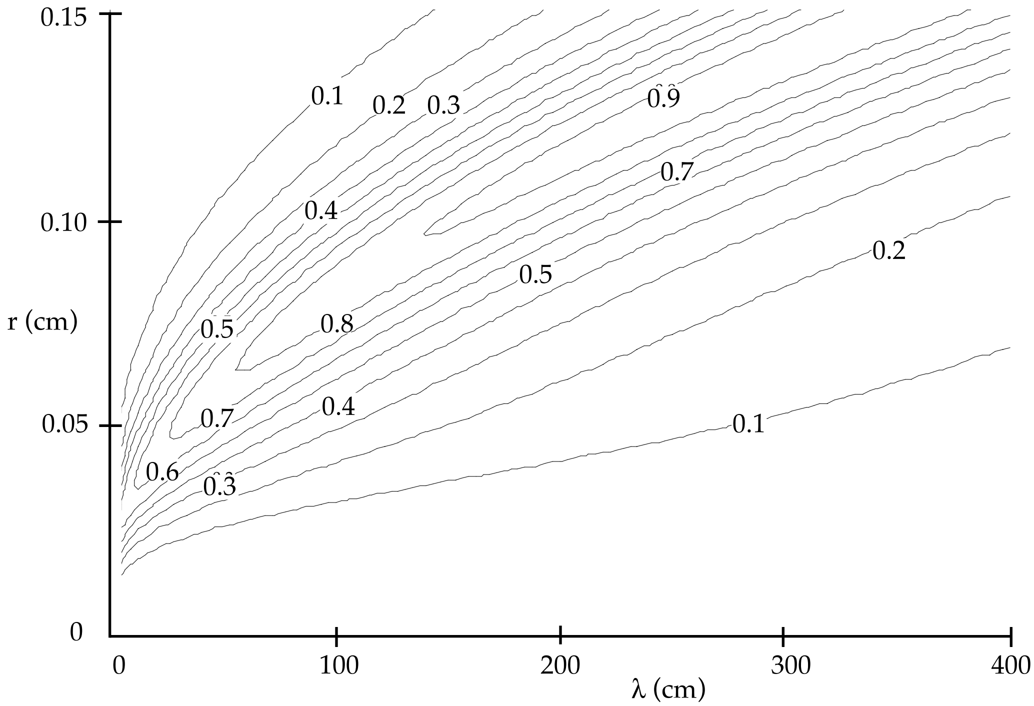

Fig. 2. The efficiency of the electrical analog of an inertance pulse tube as a function of the length, λ, and the radius, r, of the inertance tube. The lines are contours of constant efficiency.

of efficiency. Clearly, the efficiency is greatest for λ and $r = \infty$. However, the efficiency can be quite close to the optimum value (1.0 for this case) even with finite values of λ and r. For any given value of r, the value of λ that gives the best efficiency is:

$$\lambda_{opt} = \frac{\pi r^2}{\rho\, \omega^2 C_2} \frac{k}{\sqrt{k^2+1}}\ , \tag{11}$$

where ω is the angular frequency, $k = \omega\, \rho\, r^2/8\mu$ and $C_2 = C_{regen/2} + C_{p\ tube} + C_{inert/2}$. For this value of λ:

$$\text{efficiency}\left(\lambda_{opt}\right) = \frac{1}{1 + 2\, \omega\, C_2\, R_1\left(\sqrt{k^2+1} - k\right)}\ . \tag{12}$$

For $k >> 1$, $\lambda_{opt} = \pi\, r^2/(\rho\, \omega^2\, C_2)$ or

$$L_{opt} = 1/(\omega^2\, C_2) \quad \text{by eq. (8)}, \tag{13}$$

and the efficiency = 1. For the case analyzed in Fig. 2 and for $r = 0.13$ cm, then $k = 11.5$, $\lambda_{opt} = 265$ cm and the efficiency = 0.94.

By way of comparison, for the case where the inertance tube is replaced by an orifice (a simple resistance in the electrical analog) of optimum value, the efficiency becomes

$$\text{efficiency}\left(\text{orifice}_{opt}\right) = \frac{1}{1 + 2\, \omega\, C_2\, R_1} \tag{14}$$

which reduces to an efficiency = 0.42 for the same parameters used above.

This is only suggestive of the way that the inertance tube benefits a pulse tube cooler. Many of the features of a cooler cannot be modeled by such an analog: the presence of temperature gradients in the actual cooler are one of the obvious effects that an electrical analog can't deal with. Nevertheless, it is clear that this treatment captures the primary way the inertance tube yields the benefits it does.

We have adapted our ARCOPTR computer model[4] to include an inertance tube. This allows us to easily and quickly study the effect on a given pulse tube design of changing the orifice to an inertance tube. This modification to the model was readily implemented since the model already contained a detailed analytic solution for flow in the hot heat exchanger; it was only necessary to take out the screen mesh, make the heat exchanger long and thin, and it thus becomes an isothermal inertance tube. This solution includes all the effects that arise for lengths comparable to the acoustic wavelength. In electrical terms, it treats the inertance tube as a lossy transmission line.

It was also necessary to treat the effects of turbulent flow since the velocities tend to be rather high in the inertance tube. To do this, we used a Reynolds number, N_δ, that is appropriate for oscillating flow[5]:

$$N_\delta = \frac{q\,\delta_v\,\rho}{\mu} \tag{15}$$

where δ_v is the boundary layer thickness (or Stokes length): $\delta_v = \sqrt{\frac{\mu\,\tau}{\pi\,\rho}}$ where τ is the period of oscillation (q, ρ and μ are defined for Eq. 4). This is appropriate because the transition to turbulent flow occurs first in the boundary layer; it is the boundary layer dimension rather than the tube diameter that determines the critical Reynolds number for oscillatory flow. The friction factor for turbulent flow is determined from the Blasius eq.:

$$f = C\,N_D^{-1/4} \tag{16}$$

where N_D is the Reynolds number using the tube diameter, D: $N_D = qD\,\rho/\mu$, and C is a constant near 0.1.

Three model pulse tube coolers

Orifice versions. We have modeled three orifice pulse tube coolers of rather different sizes; their dimensions are shown in Table 1. The net cooling power is the sum of the enthalpy flow at the cold end of the pulse tube and the various losses in the system (the residual enthalpy flow from the regenerator and the conduction losses from the regenerator wall and matrix and from the pulse tube wall). The efficiency is the ratio of this net cooling power to the compressor PV work. In each case the orifice setting used was that which maximized the efficiency.

Table 1. Orifice pulse tube coolers, modeled for T_{hot}=300 K and T_{cold}=80 K freq. =55 Hz, Pressure =2.0 x10^6 Pa, pressure ratio in pulse tube =1.20.

	Regenerator				**Pulse Tube**		**Orifice**	**Cooling Power**	**Compress. PV work**	**Efficiency**
	ID (cm)	L (cm)	Mesh (/in)	Wire (cm)	ID (cm)	L (cm)	(kg/s Pa)	(W)	(W)	
Pulse Tube 1	0.615	5.00	400	.0025	0.87	2.01	8.4	0.82	64.7	1.27%
Pulse Tube 2	1.545	4.68	400	.0025	1.10	3.00	21.8	3.37	93.2	3.63%
Pulse Tube 3	3.77	4.30	400	.0025	1.55	5.02	90.0	15.95	333.8	4.78%

Table 2. The same pulse tube coolers as in Table 1 with the orifice replaced by an inertance tube in the model calculation; pressure ratio in pulse tube =1.20.

	Inertance Tube	Cooling Power	Compress. PV work	Efficiency	Eff. (Inert. tube) / Eff. (Orifice)
	length x dia.(cm)	(W)	(W)		
Pulse Tube 1	100 x 0.104	0.98	48.3	2.02%	1.59
Pulse Tube 2	250 x 0.22	4.72	81.4	5.79%	1.60
Pulse Tube 3	400 x 0.58	17.9	215	8.34%	1.74

Inertance tube versions. We then replaced the orifice of each cooler with an inertance tube that was chosen to give the best cooling performance. The results of this model calculation are shown in Table 2. For all three pulse tubes the inertance tube increases the efficiency by about a factor of 1.6 to 1.7 under similar operating conditions. Part of the increase in efficiency is brought about by an increase in cooling power but most of the increase comes from a reduction in the required compressor PV work. This is one of the main benefits of an inertance tube, that it significantly lowers the compressor PV work required for the same system with an orifice.

Comments on modeling. Figure 3 shows the results of more detailed calculations for pulse tube 1. For a wide range of inertance tube lengths, from 60 cm to 150 cm, there is a value of inertance tube radius, r, that brings the efficiency to about 2%. The values of λ/r^2 (which is proportional to the inertance) at which these maximum efficiencies occur are clustered around 9000 cm^{-1} for this example. For a given value of λ, however, the efficiency is sharply peaked at a particular value of r.

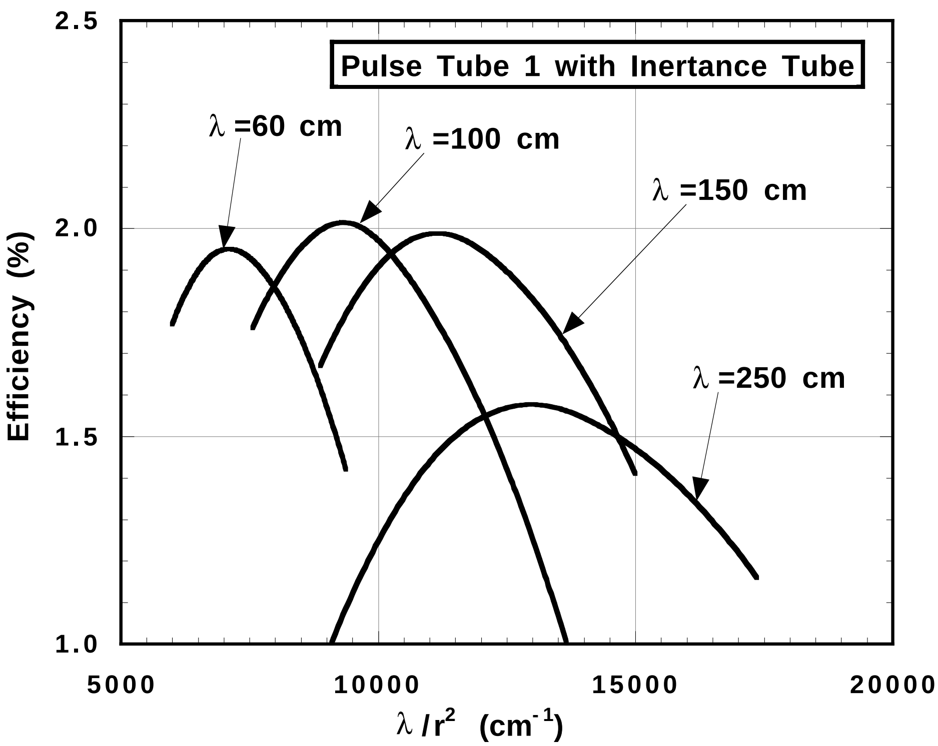

Fig. 3. Efficiency of pulse tube 1 from model calculation for different values of inertance tube length, λ, as a function of λ/r^2, where r is the inertance tube radius.

Table 3. Experimental orifice pulse tube cooler, with T_{hot} = 303 K, freq. = 65 Hz and pressure = 2.27 x10^6 Pa. No heat is applied to the cold heat exchanger.

	Regenerator				**Pulse Tube**		**Press. ratio in PT**	**Calculated compressor PV work**	**T(cold)**
	ID (cm)	L (cm)	Mesh (/in)	Wire (cm)	ID (cm)	L (cm)		(W)	(K)
Experimental pulse tube	1.54	6.73	400	.0025	1.27	5.69	1.13	113	111

EXPERIMENTAL TESTING

We have performed a similar experimental comparison by measuring the lowest temperature reached by a laboratory pulse tube with an orifice and then replacing the orifice with an inertance tube and measuring the change in lowest temperature reached.

Orifice version.

Table 3 gives the dimensions of the experimental pulse tube cooler we used and the results of the measurements with the orifice set to give optimum cooling. The pressure ratio in the pulse tube is the maximum that we can obtain from our compressor for this cooler. The calculated compressor work comes from a fit to the experimental data by our model. The model determines the mass flows through the various parts of the cooler in order to match the experimental pressures at the compressor, the pulse tube and the reservoir. From this information the model calculates the PV work of the compressor, assuming adiabatic compression.

Inertance tube version.

Table 4 shows the results of measurements on the same experimental cooler with the orifice replaced by two different inertance tubes. Again, the pressure ratio in the pulse tube is the maximum that we can obtain from our compressor. The fact that the pressure ratio in the pulse tube is higher for the inertance tube versions than for the orifice version is one of the advantages of an inertance tube. For inertance #1 the minimum temperature, 118 K, isn't quite as low as that with the orifice, 111 K. Note,

Table 4. The same experimental pulse tube cooler as Table 3 but with two different inertance tubes replacing the orifice; T_{hot} = 303 K, freq. = 65 Hz and pressure = 2.3 x10^6 Pa. No heat is applied to the cold heat exchanger.

	Inertance Tube	**Press. ratio in PT**	**Calculated compressor PV work**	**T(cold)**
	length x dia.(cm)		(W)	(K)
Inertance 1	105 x 0.17	1.17	82	118
Inertance 2	226 x 0.28	1.17	76	92

however, that the calculated compressor PV work is only 73% of that for the orifice version. For inertance #2 the minimum temperature, 92 K, is considerably lower than it is with the orifice, and the compressor PV work is even lower, at 67% of that for the orifice version. Clearly, the inertance tubes provide considerable benefit in both cases.

CONCLUSIONS

The advantages of employing an inertance tube in the design of a pulse tube cooler have been shown by a simplified electrical analogy, by detailed computer modeling and by experimental measurements.

The electrical analogy reveals that the optimum inertance (the electrical inductance) obeys a resonance condition (eq. (13)) in terms of the volume in the middle of the cooler (half the regenerator volume plus the pulse tube volume plus half the inertance tube volume). This shows that an optimum inertance tube depends primarily on the volumes in the system (except the compressor volume) and not on the impedance of the regenerator.

Computer modeling for three different size pulse tubes shows that a large benefit of about a factor of 1.6 can be gained in the cooling efficiency for a pulse tube cooler with an optimized inertance tube over one with an optimized orifice. It also shows thaat there is a sharp maximum in cooling efficiency as a function of r for a fixed value of λ. However, for a wide range of λ there are values of r that bring the efficiency to about the same maximum value.

Experimental comparisons of a pulse tube with an orifice and with two different inertance tubes likewise show substantial benefits either in the lowest temperature reached, the compressor PV power needed, or both, when employing an inertance tube.

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